Deterministic Almost-Linear-Time Gomory-Hu Trees

Amir Abboud (Weizmann Institute of Science),
Rasmus Kyng (ETHZ),
Jason Li (CMU),
Debmalya Panigrahi (Duke),
Maximilian Probst Gutenberg (ETHZ),
Thatchaphol Saranurak (UMich),
Weixuan Yuan (ETHZ),
Wuwei Yuan (ETHZ)

FOCS 2025

December 16, 2025

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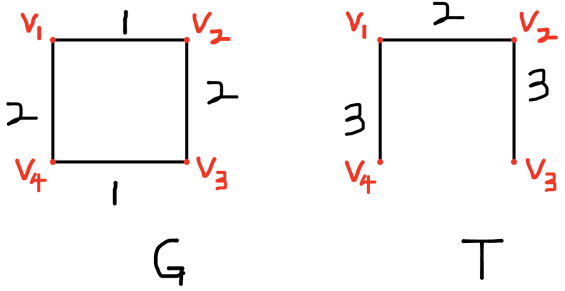
- Given graph $G = (V, E, w)$, compute tree T on same vertex set s.t. $\text{mincut}_G(s,t) = \text{mincut}_T(s,t)$ for all $s, t \in V$.
- T is known as **Gomory-Hu tree**.



Related Works

- Max-flow takes rand. [CKLPPS'22] / deter. [BCKLPPSS'23][BCKLMGS'24] $m^{1+o(1)}$ time.
- $\tilde{O}(\cdot)$ hides polylog(n) factors.

Reference	Rand./Deter.	Time
Gomory-Hu'61	deter.	$nm^{1+o(1)}$
Abboud-Krauthgamer-Li-Panigrahi-Saranurak-Trabelsi'21	rand.	$\tilde{O}(n^{2.875})$
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- Assume graphs are **unweighted**.
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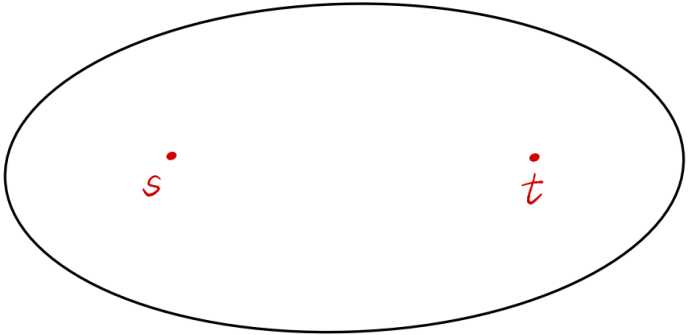
Arbitrary $s, t \in V$



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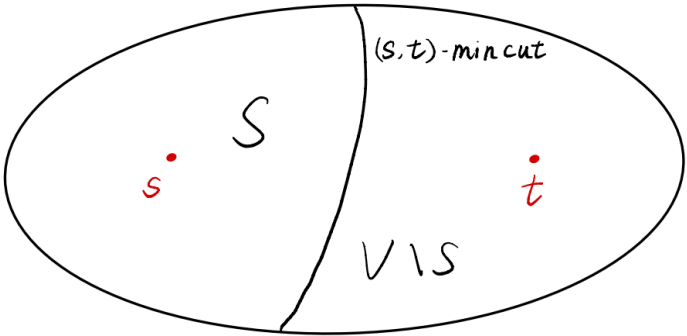

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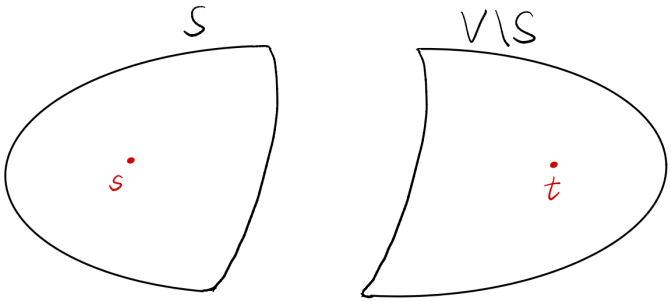

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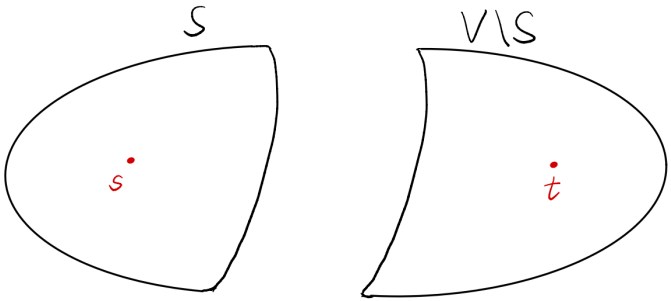

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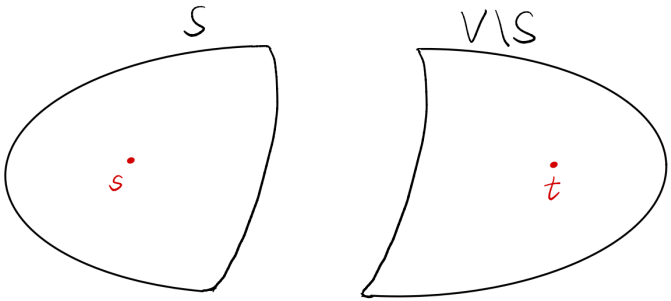

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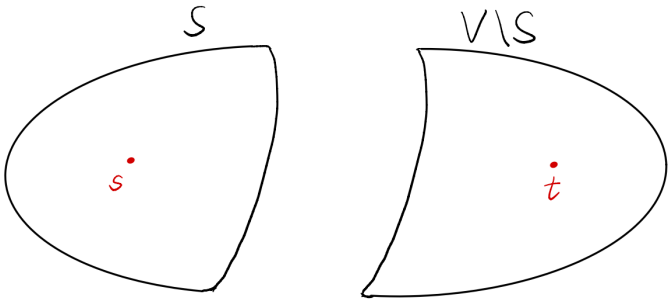

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- Unbalanced decomposition $\implies$ large recursion depth $\implies$ superlinear complexity

Isolating Cuts

- **Isolating cuts** (Li-Panigrahi'20)
  Input: G, vertices $P = \{v_1, \ldots, v_l\} \subseteq V$
  Output: disjoint vertex sets $S_1, \ldots, S_l$, such that S_i is the inclusion-minimal min $(v_i, P \setminus v_i)$ cut.

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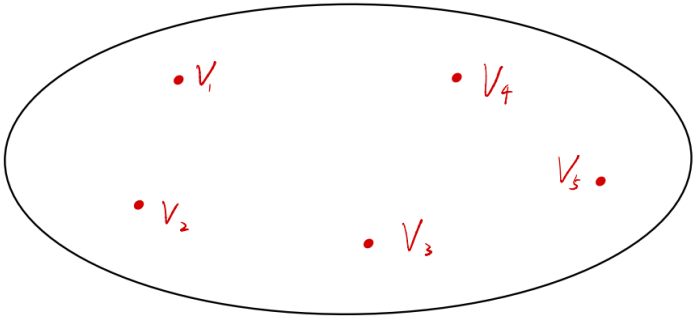

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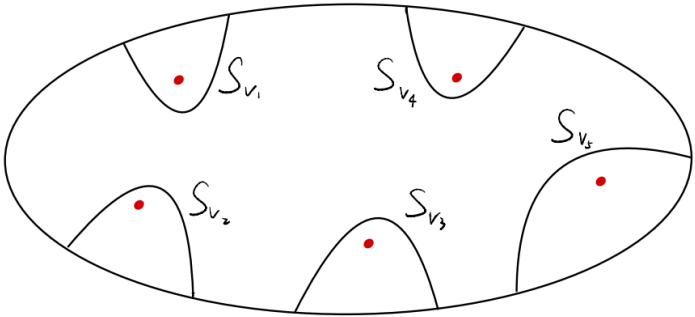

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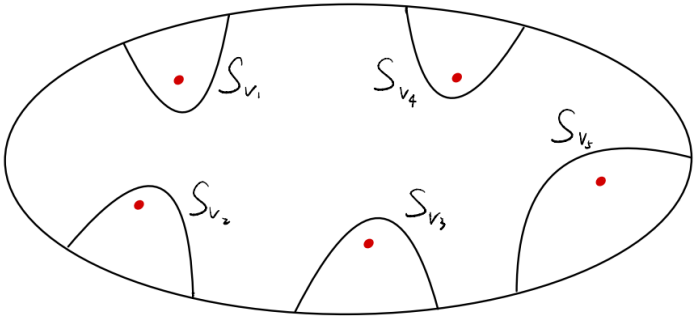

- Time cost: $\tilde{O}(T_{maxflow}(m)) = m^{1+o(1)}$.

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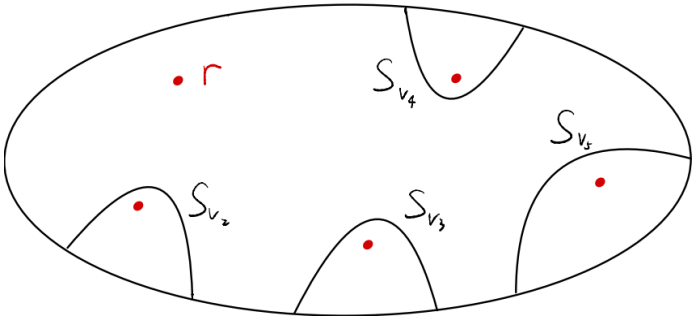

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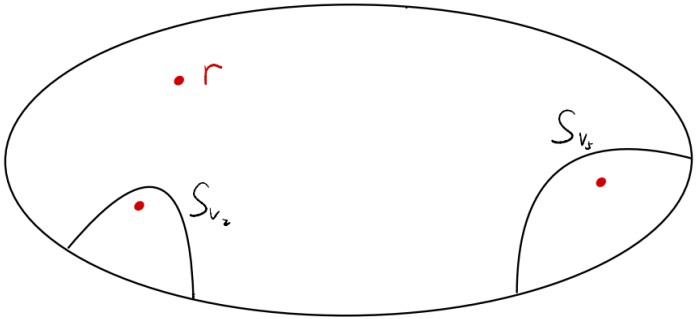

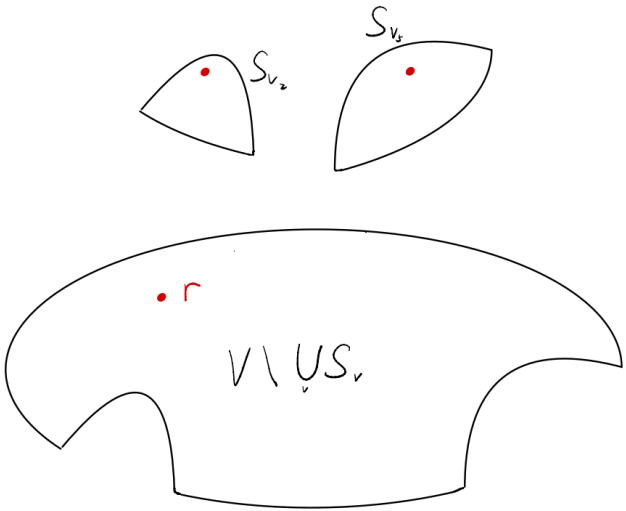

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How to select r and P?

Randomized Almost-Linear-Time Algorithm (ALPS'23)

- Sample a pivot r u.a.r.
- For $i = 1, 2, \ldots \lfloor \log n \rfloor$, sample vertex set P with $|P| = 2^i$ vertices u.a.r.

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In expectation, one of the decompositions is balanced.

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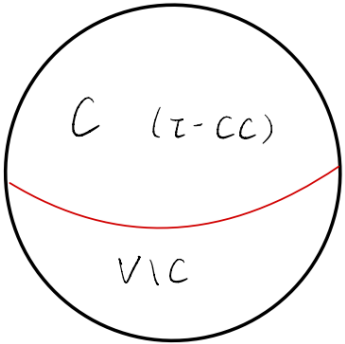

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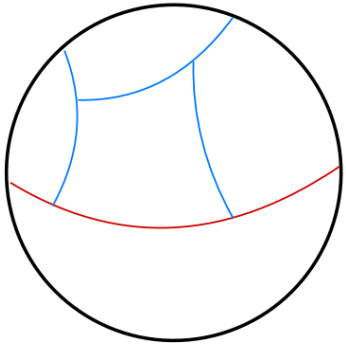

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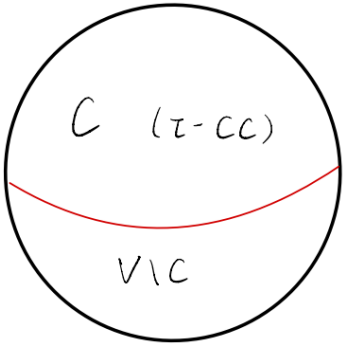
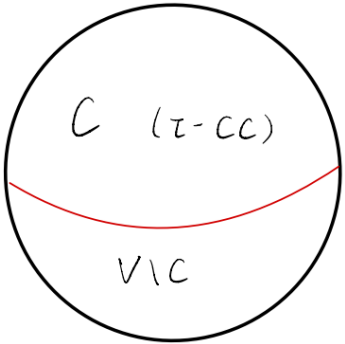


Pick an arbitrary $r \in C$ as the pivot.

Case 1: C is small



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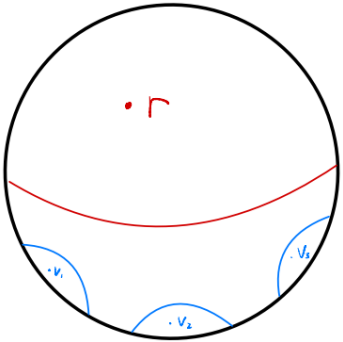

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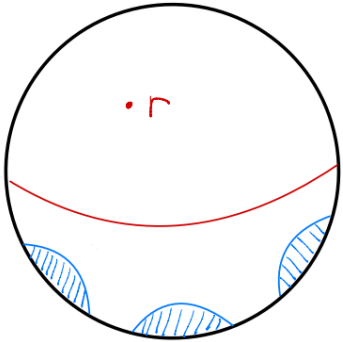

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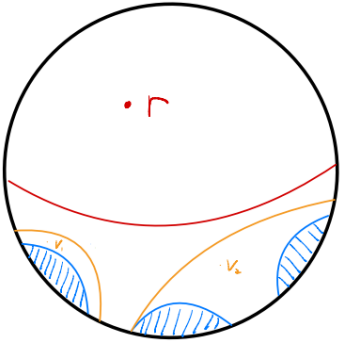

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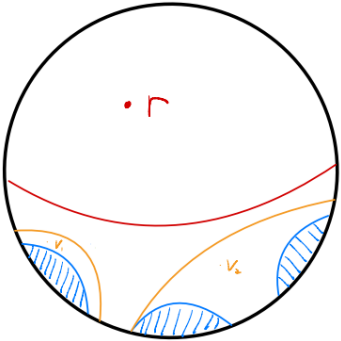

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We get a collection of (inclusion-minimal) (r, v)-mincuts. Their union is $V \setminus C$

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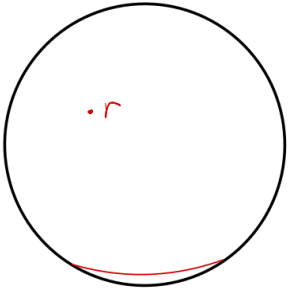

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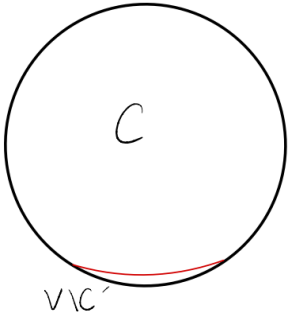

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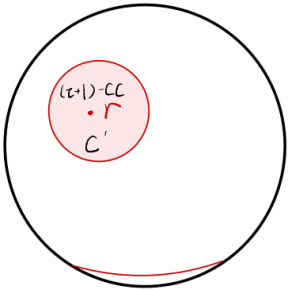

1. Start from some $A \leftarrow C \setminus C'$
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- **Guide trees to SSMC**

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A Simple and Fast Reduction from Gomory-Hu Trees to Polylog Maxflows.
by Maximilian Probst Gutenberg, Rasmus Kyng, Weixuan Yuan, Wuwei Yuan

A Simple Deterministic Reduction From Gomory-Hu Tree to Maxflow and Expander Decomposition
by Maximilian Probst Gutenberg and Weixuan Yuan

Summary: Gomory-Hu Trees

- Deterministic $m^{1+o(1)}$-time algorithm for weighted graphs. [**This Work**]
  Also:
  - Simple randomized $\tilde{O}(T(n,m))$-time algorithm for unweighted graphs. [KP**YY**'25]
  - Simple deterministic $\tilde{O}(T(n,m) + T_{ExpDecomp}(n,m))$-time algorithm for unweighted graphs. [P**Y**'25]
- Open problems:
  - Simple algorithms for weighted graphs.
  - $\tilde{O}(T(n,m))$-time algorithm for weighted graphs.
  - Efficient dynamic algorithms.
    [Kenneth-Mordoch and Krauthgamer'25]: Deterministic fully-dynamic algorithm with $n^{3/2+o(1)}$ worst-case update time.
  - Generalize Gomory-Hu tree to hypergraphs, element connectivity, and vertex connectivity.

Thanks!